Logic and science: science and logic

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Ole Hjortland [2017] lists the following tenants of "anti-exceptionalism about logic":

Logic isn't special. Its theories are continuous with science; its method continuous with scientific method. Logic isn't a priori, nor are its truths analytic truths. Logical theories are revisable, and if they are revised, they are revised on the same grounds as scientific theories.

Those of us who were trained in logic, and work in it, do presumably think (or hope) that logic is special. Why would we devote so much energy to something that is ordinary, less than special? The physicist presumably thinks (or hopes) that physics is special; the biologist that biology is special, etc.

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Of course, this is not what is meant. The anti-exceptionalist holds that logic is *not different* from other respectable forms of inquiry, *science* in particular.

Well, every form of inquiry is different, in crucial ways, from every other. Is there enough in common between logic and a typical science in order to have a sufficiently clear thesis of anti-exceptionalism to defend or reject? Indeed, one must clarify a number of things before we can assess anti-exceptionalism. There are a lot of balls in the air. To switch metaphors, there are lot of moving parts in this discussion.

The notions of a priority and analyticity are, of course, vexed. Some, following Quine, hold that neither of these marks an interesting or important distinction. Others argue about what the distinctions come to, and how they are used. So that matter must be addressed.

The anti-exceptionalist also says that logical theories are *revisable*.

Well, any theory can be revised, if a better one comes along.

Perhaps the anti-exceptionalist claims that logic, or the correct logic, is not known with absolute certainty. It is defeasible, in the same sense that scientific theories are. The anti-exceptionalist further holds that logical theories are revised *on the same grounds* as scientific theories are revised.

Well, what are those grounds?

The anti-exceptionalist says that the method of logic is "continuous" with "scientific method".

Well, what is scientific method? And, while we are at it, what is it for one inquiry to "continuous" with another one?

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The issues Aside on Aristotle

On this last matter, we might get some help, by way of analogy, from Aristotle. In Book 5 of *Physics*, he says that two things are "contiguous", or "in contact", if they are next to each other in such a way that nothing can go between them: "Things are said to be in contact when their extremities are together" (226b21).

Think of a pair of adjacent books on a tightly packed shelf.

The issues Aside on Aristotle

Aristotle goes on to define continuity, as a relation between two objects:

The continuous is a subdivision of the contiguous: things are continuous when the touching limits of each become one and the same and are, as the word implies, contained in each other: continuity is impossible if these extremities are two. This definition makes it plain that continuity belongs to things that naturally in virtue of their mutual contact form a unity. (227a10-15)

The issues Aside on Aristotle

So the books on the shelf are not continuous, since each retains its own boundaries—each maintains its own "unity".

By way of analogy, then, two disciplines (say logic and science) are continuous with each other if one cannot maintain sharp boundaries between them—if one cannot tell where one ends and other other begins, or if they, together, form a kind of unity.

This seems to capture at least some of the spirit of anti-exceptionalism.

The details are another story.

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Apparently, the anti-exceptionalist puts a lot of weight on *how* various scientific theories and logics are revised.

Are the theories of all of the sciences revised on the same kinds of grounds?

And do all of the sciences have the same method?

If the answer to either of these questions is "no", then which are the scientific theories to which logic is continuous? And which of those sciences use the same method as logic does? In short, what are the "sciences" that are relevant here?

Timothy Williamson [2017], a noted anti-exceptionalist, counts *mathematics* as a science.

Are mathematical theories are ever revised? At least on the present scene, Euclidean geometry and all of the various non-Euclidean geometries are legitimate mathematical theories, not subject to revision. As Alberto Coffa [1986, 8, 17] once put it (with characteristic wit): During the second half of the nineteenth century, through a process still awaiting explanation, the community of geometers reached the conclusion that all geometries were here to stay ... ".

There is, of course, a compelling question as to which mathematical theory is best applied in a given context, say which geometry gives the best theory of physical space, but that is not a case of *mathematics* being revised.

The issues mathematics

It is generally agreed, even by those who support monism, that the various formal logics—classical, intuitionistic, paraconsistent, paracomplete, ...—are themselves legitimate pieces of *mathematics*, in the same sense that the various geometries are legitimate as mathematics.

The debate is over which (if any) of those logics is the, or a, correct account of *validity* or *logical consequence*.

That is where our present concern lies.

There is an enterprise of seeking and developing a *foundation* for mathematics, a single theory in which all others can be defined.

One can ponder revising the foundation, in the sense of using a different theory—set theory or category theory perhaps—to play that role.

And, given a particular proposed foundation, say set theory, one can ponder whether adding some new axioms—say those about large cardinals—enhances the foundational enterprise.

Such matters have been treated, in detail, in the foundations literature (see Feferman, Friedman, Maddy, and Steel [2000]). But, at least prima facie, that seems different from how, say, physics was revised due to relativity and quantum mechanics.

Williamson also counts the *social sciences*, such as psychology, as within the purview of his anti-exceptionalism.

For Frege, a "science" is *any* organized body of knowledge. So history counts as a science (or can, once its truths are sufficiently organized).

Do all of these enterprises—mathematics, physics, chemistry, psychology, sociology, economics, history—share enough methodology for us to even ask if logic has that methodology, too?



To be sure, all of the sciences make essential use of deduction, and so does logic, but that is somewhat unhelpful. Deduction, or at least deductive validity, is among the special topics of logic itself.

There is one more (very) large, and (very) vexed, batch of questions and issues that have to be settled before one can assess anti-exceptionalism. Namely, what is logic about? Or to be even more blunt, what is logic?

We can perhaps agree that the goal of a logic is to characterize or codify *validity*, or *logical validity*, or *logical consequence*.

But what is that (or what are those)?

There is nothing but controversy over what those notions are.

There is also heated debate as to what they are relations of: sentences of natural language, forms, sentences of an ideal language, propositions, ...

Alfred Tarski's [1935, 409] celebrated "On the concept of logical consequence" opens

The concept of logical consequence is one of those whose introduction into the field of strict formal investigation was not a matter of arbitrary decision on the part of this or that investigator; in defining this concept, efforts were made to adhere to the common usage of the language of everyday life. But these efforts have been confronted with the difficulties which usually present themselves in such cases. With respect to the clarity of its content the common concept of consequence is in no way superior to other concepts of everyday language. Its extension is not sharply bounded and its usage fluctuates.

The issues What is logic?

Any attempt to bring into harmony all possible vague, sometimes contradictory, tendencies which are connected with the use of this concept, is certainly doomed to failure. We must reconcile ourselves from the start to the fact that every precise definition of this concept will show arbitrary features to a greater or less degree. Tarski seems to suggest here that we are confronting a quasi-empirical question concerning the meaning of the English phrase "logical consequence", or the English word "valid", perhaps as used by professional logicians, or by certain experts, to which competent speakers are prepared to defer (or ought to be prepared to defer).

Or else one might think that one of the phrases somehow picks out a certain *concept*, or *relation*, and the dispute is over that very concept or relation.

Perhaps logical consequence is something of a natural kind. And then the debate begins: some say that the concept is a certain way; others say that it—that very concept—is some other way.

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Shapiro [1998] and Shapiro [2014, Chapter 2, $\S2$] lists a number of different senses in which one can say that a sentence or proposition is a logical consequence of a set of sentences or propositions. No doubt there are others. The parenthetical names are only meant to be suggestive, not historically accurate. And we do not assume that all of these notions are distinct.

The following appears in Aristotle's Prior Analytics, Book 1, Chapter 2:

A deduction is a discourse in which, certain things having been supposed, something different from the things supposed results of necessity because these things are so. By "because these things are so", I mean "resulting through them" and by "resulting through them", I mean "needing no further term from outside in order for the necessity to come about. This is a modal notion of consequence. A more contemporary modification of Aristotle's notion is that ϕ is a logical consequence of Γ just in case:

(1) It is not possible for every member of Γ to be true and Φ false (Aristotle).

This captures a common slogan that validity is the necessary preservation of truth. Spelling this out in a now common framework leads to:

(2) ϕ holds in every possible world in which every member of Γ holds.

There are also linguistic, or *semantic*, characterizations of consequence:

(3) ϕ holds in every interpretation of the language in which every member of Γ holds (Tarski [1935]).

(4) The truth of the members of Γ guarantees the truth of ϕ in virtue of the meanings of the terms.

The issues What is logic?

(5) The truth of the members of Γ guarantees the truth of ϕ in virtue of the meanings of a special collection of the terms, the "logical terminology" (Tarski [1935]). (6) There is no uniform substitution of the non-logical

terminology that would render every member of Γ true and ϕ false (Bolzano [1837], Quine [1986]).

(7) The truth of the members of Γ guarantees the truth of ϕ in virtue of the forms of the sentences (or propositions).

The items (5)-(7) capture an important feature that is often, perhaps usually, thought to be part of the notion(s) of consequence, that is is formal, or that validity is a matter of form. This, too, goes back to Aristotle's *Prior Analytics*.

The issues What is logic?

And there are epistemic/normative characterizations, since, after all, logic surely has something to do with (deductive) reasoning:

(8) It is irrational to maintain that every member of Γ is true and to fail to maintain ϕ .

(9) There is a deduction of ϕ from Γ by a chain of legitimate, gap-free (self-evident) rules of inference (Aristotle, Leibniz [1686], Frege [1879]).

There is a tradition, going back to antiquity, that insists that ϕ is not a logical consequence of Γ unless Γ is somehow *relevant* to ϕ . This matter, of course, is hotly disputed, and always has been.

Another slogan is that logic is *absolutely general*, and *topic neutral*. It applies to any and all discourses about any and all things. This, too, is hotly debated (Shapiro [2014]).

We do not claim that all of these intuitive notions are completely distinct from each other. Some seem to be related to others, and some are developments, or represent theories, of others.

There may be a tight relationship between the modal notions (1-2) and at least some of the semantic ones (3-7). This depends on the extent to which the modality invoked in the modal notions is to be understood in terms of the *meaning* of the constituents of the sentences or propositions: Are the "possibilities" in question, in the modal conceptions, to be understood as "interpretations" of the language or of part of the language?

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Our first conclusion is that it is entirely unclear just what anti-exceptionalism is, and it remains unclear until all, or at least many, of the foregoing questions and issues are revolved.

Which sciences are we talking about?

What are the relevant bits of scientific and logical methodology that we are comparing?

And, perhaps most important of all, what is logical consequence? What is the logician giving a theory of?

We propose to briefly examine the work of three avowed anti-exceptionalists: Hjortland [2017], Williamson [2017], and Graham Priest [2014], [2016]; and two others in the "naturalist" tradition who turn their attention to matters logical: Penelope Maddy [2002], [2007, Part III] and John P. Burgess [2015, Chapters 1-2]. Hjortland, Williamson, and Priest explicitly discuss the question of how one rationally chooses among different logics, or, in other words, how one settles on one or more of the various candidates: classical, intuitionistic, paraconsistant, paracomplete, ...

The overall methodology is described as "abductive", an inference to the best explanation. And, in broad terms, they all list essentially the same criteria that are used to decide between rival "theories"—rival logics.

These are the ones invoked in standard discussions of theory choice in elementary philosophy of science texts: adequacy to the "data" or "evidence", along with simplicity, unifying power, not being *ad hoc*, etc.

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But the agreement between our anti-exceptionalists ends there. When it comes to abduction—inference to the best explanation—they differ, rather sharply, on what the *explananda* are, and, probably what is the same thing, they differ on the what the "data" are (or what the "evidence is") when choosing between logics.

That is, our anti-exceptionalists do not agree on what it is that the logic is supposed to be *adequate to*, or what it is supposed to *explain*.

The reason, it seems, is that they give different accounts of what validity or logical consequence is.

Some anti-exceptionalists and naturalists Williamson

Williamson gives a broadly Tarskian account of logical consequence, roughly along the lines of (6) above.

Consider an interpreted, but formalized (or regimented) language, and assume that its first-order quantifiers are absolutely unrestricted—they range over everything.

Begin with a given sentence. Replace each non-logical term with a variable of appropriate type, and bind that variable with a universal quantifier. The original sentence is logically true just in case the result is true.

Validity is defined in terms of logical truth

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Some anti-exceptionalists and naturalists $\ensuremath{\mathsf{Williamson}}$

So, for Williamson, the validity of excluded middle comes down to the truth of the following sentence in the envisioned interpreted language:

$$\forall P \forall x (Px \lor \neg Px). \tag{LEM}$$

Some anti-exceptionalists and naturalists Williamson

So, for Williamson, logical consequence has nothing special to do with language, meaning, and the like. Logic is not "meta-linguistic".

With Frege, validity concerns the most general features of reality—whether for example, (LEM) is true (recalling that his languages are "interpreted").
When it comes to the relevant "data", Williamson is consistent with his other writings: the evidence for a given logic consists of everything we know.

Of course, he does not take it as a *data point* that we know (LEM)—that would beg the question against non-classical logic. But given some agreement over what it is that we do know, he argues that classical logic scores best on the overall criteria for theory-choice.

For Hjortland, the target of logic is a number of interrelated notions, including validity, consistency, and truth.

Those, of course, are highly theoretical, and so perhaps there is nothing that plays the role of observation (or "data") in science.

Priest [2016, $\S2.5$] gives a similar but different list of related topics (validity, consistency, negation, meaning ...) as the *explananda* of logic. But he adds

In the criterion of adequacy to the data, what counts as data? It is clear enough what provides the data in the case of an empirical science: observation and experiment. What plays this role in logic? The answer, I take it, is our intuitions about the validity or otherwise of vernacular inferences.

Priest holds that this "data" is corrigible, and subject to revision in light of theory.

In this respect, logic is not different from natural science: observation is theory-laden; intuitions surely are, too.

In other work, for example, Priest admits the pull of disjunctive syllogism, as an "intuition", but rejects it for theoretical reasons, as it is incompatible with his theory of truth (and other things) (see, e.g., Priest [2006]).

Against Williamson, then, Hjortland and Priest have it that logic *is* "meta-linguistic", matters of language and meaning are involved.

For Priest, logic concerns the meaning of key logical terms, such as those expressing negation, disjunction, and the like.

For Hjortland, the consequence relation can be restricted to various languages.

And for both, truth, and truth-preservation, are among the target notions, and those are, arguably, meta-linguistic (at least as they understand and invoke those notions).

As noted, Williamson takes logic to be about the most general truths, as formulated in an interpreted language whose first-order variables range over absolutely everything. As such, logic is not about language; its truths are not analytic, etc. Also as noted, Williamson accepts the truth of (LEM),

 $\forall P \forall x (Px \lor \neg Px),$

where, again, the variable x ranges over (absolutely) all objects and P ranges over all properties (or all predicates).

It is well-known that some intuitionistic theories have, as theorems, statements that are in the form of the negation of (LEM).

For example, intuitionistic analysis and smooth infinitesimal analysis (as well as its stronger cousin synthetic differential geometry) prove that

$$\neg \forall x (x = 0 \lor x \neq 0).$$

This contradicts (LEM) above.

For intuitionistic analysis, let P be the property of being an intuitionistic real number that is identical to zero, and for smooth infinitesmal analysis, let P be the property of being a nilsquare that is identical to zero.

Another relevant example is Heyting arithmetic with Church's thesis. In that case, the relevant instance of P is the "self-halting property" of being an intuitionistic natural number x that is the index of a Turing machine that halts when given x as input.

As far as we can tell, Williamson has just two options here. One is to reject these intuitionistic theories as contradictory, and thus incoherent.

Intuitionistic mathematics would be a casualty of the holistic, abductive conclusion to accept classical logic as an account of the most general truths.

The conclusion here would be of-a-piece with Williamson's explicit rejection of an unrestricted truth predicate on the same grounds, that classical logic that wins the holistic, abductive battle.

In the present case, the conclusion comes despite the fact that the intuitionistic theories are respectable intellectual endeavors, pursued by mathematicians whose credentials are beyond question, and the results appear in mainstream mathematics outlets.

As the saying goes, one person's *modus ponens* is another's *modus tollens*. A different theorist might conclude that the intuitionistic theories are too important to be given up. In that case, it is (LEM) that must go—still sticking to the Williamson program that logic is about the most general truths.

We do not have any more to say on how this holistic battle should be waged.

The other option for Williamson is to insist that in the intuitionistic theories, the logical terminology does not mean what it does in the classical ones.

In particular, the intuitionistic negation and disjunction are not the same as the connectives that appear in (LEM).

This, of course, is a familiar line, held by thinkers as diverse as Carnap, Quine, and Dummett (see Shapiro [2014, Chapter 4]).

It invites a debate over what the meaning of the logical terminology, in various discourses, is, and how we decide what counts as the same or different meaning.

Williamson insists that these "meta-linguistic" matters are foreign to logic, or to the choice of which logic is correct. It seems, however, that they are not so foreign, at least if one is to take intuitionistic mathematrics seriously.

We have no more to say on how *this* battle is to be fought.

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Recall part of the passage from Priest [2016, §2.5] above:

It is clear enough what provides the data in the case of an empirical science: observation and experiment. What plays this role in logic? The answer, I take it, is our intuitions about the validity or otherwise of vernacular inferences.

We do not know of any other "science" for which the theorist's own "intuitions" about something serve at the "data", or for which those "intuitions" are the main *explananda* of the abductive methodology of the science.

Within psychology, there are projects of determining folk theories of various things, such as time and space. But in those enterprises, the researcher does not consult his or her own "intuitions" of those things.

The data are obtained in familiar ways, using standard empirical methodology, to rule out the bias of the researcher and other know failings.

Moreover, it is not assumed that these folk theories are, or should be, made rigorous, nor that they hold in any and all situations, observed or not. And the "data" of those enterprises are not up for correction in light of theory.

And, of course, these folk theories do not provide data for physics, which is concerned with the actual nature of time and space, and the like. Some anti-exceptionalists and naturalists $\ensuremath{\mathsf{Priest}}$

Perhaps, by way of analogy, there is room for a "folk theory" of validity, but that should not be confused with the nature of validity itself. Presumably, it is the latter that is the target of logic.

Priest [2016, $\S2.5$] goes on to give us some examples of what he takes the "data" for logic to be:

inferences such as the following strike us as correct:

John is in Rome. If John is in Rome he is in Italy. John is in Italy.

John is either in Rome or in Florence. If John is in Rome he is in Italy. If John is in Florence he is in Italy. John is in Italy.

and the following strike us as invalid:

John is either in Rome or in Florence. John is in Rome.

If John is in Rome he is in Italy. John is not in Rome. John is not in Italy.

Any account that gets things the other way around is not adequate to the data.

To be sure, our experience with some students, and studies like the Wason Selection Task suggest that this "data" is not shared by everyone, but perhaps we can agree, in such cases, that theory has corrected this "data".

Within linguistics, semanticists do take intuitive inferences like those cited here as part of the "data" for accounts of the meanings of natural language expressions (or expressions "in the vernacular" as Priest puts it).

Judgements of felicity are also among the consulted data. But here, too, the semanticist is not to put too much weight on his or her own "intuitions".

In [2016, §2.4], Priest says that giving "an account of validity requires giving accounts of other notions, such as negation and conditionals."

So, it seems that for Priest, valdity turns, in part, on the meaning of words like "not", "and", and "or" in natural language.

So it seems that, for Priest, logic is of-a-piece (or continuous with) empirical semantics, as pursued in linguistics (contra Williamson).

One problem with this is that the intuitve inferences cited in semantics go well beyond anything one would think of as logic. Michael Glanzberg [2015, §II.2] notes:

... natural language is permeated by entailments which strike us as evidently non-logical Here is [a] case, much discussed by semanticists

(6) a. We loaded the truck with hay. ENTAILS We loaded hay on the truck.

b. We loaded hay on the truck. DOES NOT ENTAIL We loaded the truck with hay.

This is a report of semantic fact, revealed by judgements of speakers, both about truth values for the sentences, and about entailments themselves. It indicates something about the meaning of the word 'load' and how it combines with its arguments. ...

To take one more much-discussed example, we see

(7) John cut the bread. ENTAILS The bread was cut with an instrument.

The meaning of 'cut', as opposed to, e.g., 'tear', requires an instrument . . .

Entailments like these are often called lexical entailments, as they are determined by the meanings of specific lexical items.

We presume that Priest does not include all of this linguistic information as within the purview of logic. So he must make a distinction between the data of semantics and the data of logic—the "entailments" cited by semanticists are somehow different from the "intuitions about the validity or otherwise of vernacular inferences" cited by Priest.

But what is this difference? Perhaps the idea is that the logical "intuitions" focus exclusively on the meanings of the usual range of logical terminology: negation, disjunction, the conditional, and quantifiers. This would make for a semantic account of logical consequence, along the lines of item (5) in the previous section.

However, the study of semantics reveals that natural language negations and natural language conditionals are far more complex and subtle than the negations and conditionals presented in logic texts (see, for example, Horn [1989] and Kratzer [2012]).

For similar reasons, Glanzberg [2015] argues that logic cannot be read off of the semantics of natural languages. According to Glanzberg, to get something recognizable as logic, one must first abstract from semantics, identify the logical terms, and then idealize their usage.

Perhaps the various logical and linguistic accounts are *rivals* to each other-competing accounts of the meaning of English words like "not", "if"', and "for all" (or their counterparts in other natural languages).

So each theorist puts forward an account of the very same things that the other does. On this picture, the several accounts of negation in, say, Horn [1989] stand in opposition to the theory of negation in, say, Priest [2006].

We would decide between those on general scientific grounds: adequacy to the data, simplicity, etc., assuming, of course, that the two sides can agree on what counts as data.

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Here, too, we have little more to say about how the holistic assessment is supposed to go. Clearly, the various logical accounts—whether from inferentialists or model theorists—are (much) simpler than the ones produced by semanticists, but the latter invoke a much larger and more varied (and subtle) set of data. Indeed, logicians are extremely selective in the examples they cite in favor of their accounts. Semanticists are not.

Unlike Priest, Hjortland, and Williamson, Maddy [2002] does not give a detailed description of the methodology to be used in logic:

... this approach to naturalism ... doesn't rest on any official demarcation criterion for what counts as science. The quasi-naturalist who holds to the principle 'believe only the utterances of science' might well be expected to specify what distinguishes those utterances from the rest. My naturalist takes no such global position. She is convinced by particular arguments and methods, as they come along; ... [S]he needn't espouse any global account of precisely what all these particulars have in common or any general principle on which to rule other things out. ...

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The real work comes in describing the naturalist's reaction in particular cases, in understanding what specific types of evidence are found compelling ... To a first approximation, then, my naturalist coincides with Quine's ([1975], p. 72): "The naturalistic philosopher begins his reasoning within the inherited world theory as a going concern. He tentatively believes all of it, but believes also that some unidentified portions are wrong. He tries to improve, clarify, and understand the system from within. He is the busy sailor adrift on Neurath's boat."

Maddy's Second Philosopher defines a system to have a "Kant-Frege" structure if it is composed of objects that have certain properties and relations, and that these come with certain dependency relations.

(1) psychologically, humans are so constructed that they conceptualize the world using the Kant/Frege forms of judgement and categories, and for this reason, their thinking is bound by the laws of logic; (2) objectively, the world has very general structural features that in fact correspond to the logical forms and (unschematized) categories—that is, the world consists of objects in relations, with ground/consequent dependencies between various of its aspects—and for this reason, the laws of logic are truths about the world;

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(3) humans believe the laws of logic because they are dictated by their fundamental conceptual machinery, but they come to know those laws are true by coming to know that the fundamental conceptualizations on which they are based are veridical, that is, by empirical investigation. (Maddy [2002, 69])

Note that, on Maddy's view, logic is not tied to language, or to the meanings of various words in either natural language or the languages of science. Logic concerns both the structures found in the world, and the structure of our minds as we ponder the world.

So, on this issue, she sides with Williamson, against Priest and Hjortland.

According to Maddy, we cannot read classical logic off of Kant-Frege structure. To get to something we recognize as classical logic, we must idealize, in several directions. For one thing, Kant-Frege structure allows for indeterminacies:

There is undoubtedly an apple on the table, but exactly which small bits are and aren't part of it is indeterminate. The world includes living organisms and inanimate objects, but there are indeterminate borderline cases, both kinds of objects (some primitive items) and individual objects (living things at points in the process of dying) that aren't determinately living or non-living. There are clearly tadpoles (immature creatures) and frogs (mature creatures), but the border between these is blurred. (Maddy [2007, 240-241])

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The logician also simplifies the "ground-consequence" conditional of KF-structure into the familiar material conditional.

Both of these idealizations here are likened to similar idealizations that occur throughout natural and social science. The details are subtle, and fascinating. Eventually we end up with the familiar classical logic.

One upshot of this is that logic—classical logic in this case—does not apply universally, to any and all situations we encounter (contra Williamson, and contra Frege). Like many typical scientific endeavors, the range is limited to cases where the idealizations do not distort things too much:

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... in order to move from the robust but unwieldy rudimentary logic to the power and flexibility of modern, first-order predicate logic, we must agree to steer clear of empty names and defective predicates and to adopt ... highly non-trivial idealizations. These last two take us beyond a logic that's literally true of many of the world's phenomena, but they do so for the sake of a vastly more effective instrument. The justification must be, as always, that they make it possible to achieve results that would otherwise be impossible or impractical, and that they do so without introducing any relevant distortions.

So, if classical logic is to apply to the world in a given context, several conditions must be met: there must be underlying KF-structures present; the language must be functioning properly, the names naming, the predicates classifying; the idealizations of bivalent predicates and the truth-functional conditional must be appropriate, that is, both effective and non-distorting. In such cases, our familiar logic can be trusted. (Maddy [2007, 287-288])
Some anti-exceptionalists and naturalists Maddy

So according to the Maddy's Second Philosopher, the applicability of classical logic to a given domain cannot be taken for granted, but must be checked on a case by case basis. She concludes that, so far as we can determine at present, classical logic does *not* hold in the realm of the very small:

The unpleasant conclusion is that the micro-world is not structured into things of the familiar sort; though the world does contain numerous ordinary objects, it also contains phenomena that are not so structured. Despite our scientific predisposition to see the world in these terms, our pursuit of science itself has taught us that the world is not as we expect it to be, not in all its parts.

Some anti-exceptionalists and naturalists $_{\mbox{Maddy}}$

This portion of our empirical hypothesis—that the world consists of coherent objects that move as units along continuous spatiotemporal paths—must be qualified. The world is structured into such objects at the macro-level, but at the micro-level, all current evidence suggests that it is not. (Maddy [2007, 237])

When it comes to logic, the focus of Burgess [1992], [2015, Chapters 1-2] is exclusively on *mathematics*, as a social activity. And, even here, attention is restricted to *classical* mathematics.

This is in sharp contrast with Maddy's focus on the methodology of the various natural sciences.

Burgess notes that classical mathematics is itself a *normative* endeavor, in the sense that there are standards for rigor that mathematics must meet. If a text fails to meet the standard of validity, at least up to an approximation, then it will probably not be accepted for publication in a professional journal. And if it is later discovered that a publication with an invalid inference slips through, the author will be compelled to withdraw the article, or else to correct the lapse in rigor (if possible).

For Burgess, logic is understood as a theory of rigorous proof in (classical) mathematics, in the same sense that a grammar is a theory of correct sentence construction. He writes:

Whenever a community has a practice, the project of developing a theory of it suggests itself. When the practice is one of evaluation. a distinction must be made between descriptive and prescriptive theories thereof. The former aims to describe explicitly what the community's implicit standards have been: the theory is itself evaluated by how well it agrees with the facts of the community's practice. The latter presumes to prescribe what the community's standards ought to be: the community's practice is evaluated by how well it agrees with the norms of the theory. Logic, according to almost any conception, is a theory dealing with standards of evaluation of deduction, much as linguistics deals with standards of evaluation of utterances.

The distinction between descriptive and prescriptive is familiar in the case of linguistics: no one could confuse Chomsky with Fowler. It is not less important in the case of logic. The familiar case of linguistics can help clarify a point about intuition important for logic. The data for descriptive theorizing consist of evaluations of members of the community whose evaluative practices are under investigation. (Burgess [1992, 12])

For Burgess, then, logic is classical logic. This logic is a descriptive theory of the norms of deduction implicit in the community of classical mathematicians.

Like Maddy, Burgess does not discuss the methodology of logic, but presumably, it would be similar to the methodology of linguistics, or at least that of syntax: theories of grammar. The target, in both cases, is a descriptive theory of the norms implicit in a given evaluative practice.

It seems that for Burgess, logic is not linguistic, nor meta-linguistic. It does not concern the meanings of words, except in so far as word meaning guides the evaluations behind judgements of validity in (classical) mathematics.

So here Burgess seems to agree with Williamson. But, for Burgess, the scope of logic is severely limited. It applies only to—is about—one (albeit important) activity, that of classical mathematics. Logic is not tied to the structure of the world, nor to the structure of our thought. So here Burgess disagrees with Maddy and Williamson (and Frege).

So we see that our anti-exceptionalists/naturalists present very different accounts of what logic *is*. They differ sharply on what logical consequence and validity—the targets of logic—are, and on the range or scope of logic. Priest takes logical consequence to turn on the meanings of at least the logical terms; the others do not. Matters of meaning are not particularly relevant for Williamson, Maddy, and Burgess.

Williamson (perhaps following Frege) takes logic, and logical consequence, to concern the most general truths, as formulated in an interpreted language with absolutely unrestricted quantifiers. So logic applies universally—at least to any subject matter that can be formulated in the postulated language.

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Priest also takes logical consequence to be applicable to any and all subject matters. He and Williamson note the clash between classical logic and naive principles about truth, and insist that this clash be resolved, but differ on what the best (or correct) resolution is. For Priest, the clash points toward a non-classical logic; for Williamson, it points to restrictions on the principles for truth.

Against both Priest and Williamson, Maddy and Burgess do not have logic applying universally, but they differ with each other as to *where* it applies.

Burgess restricts the scope of logic to (classical) mathematics, while Maddy restricts logic to areas of study which have a Kant-Frege structure in which the idealizations do not distort things too much. So quantum mechanics is beyond the scope (due to its apparent lack of Kant-Frege structure) and so are cases where vagueness matters (due to the idealizations).

So, all that we can conclude, on behalf of anti-exceptionalism, or naturalism, is that *if* one settles on a particular account of what logic is about—what logical consequence or validity are—*then*, depending on the account, one might be able to beat logic into the mould of one's favorite science or sciences, and, in particular, into one's prior account of how scientific theories are discovered and revised.

The particular account of logical consequence or validity would (or might) tell us that logic is out to "explain", via an abductive methodology. Or, perhaps equivalently, the particular account of logical consequence or validity would (or might) tell us what the "data" are, to which a given logic (classical, intuitionistic, paraconsistent, paracomplete, ...) has to be adequate to.

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And the logician can make the observations that in any science, the "data" are theory-laden and themselves revisable in light of theory, and she can talk about simplicity, fruitfulness, and the like. As we have seen, our anti-exceptionalists do just that. They articulate an account of what logic is, and then show how, on their favored account, logic is sufficiently like other sciences, or like their accounts of what other sciences are like, following a standard methodology of theory revision.

But the case of each of our anti-exceptionalists/naturalists is predicated on accepting a different account of what logical consequence or validity is. How does one go about adjudicating the disagreements between Williamson, Priest, Maddy, and Burgess, not to mention Hjortland, other anti-exceptionalists, exceptionalists, and pluralists?

We take it that a significant aim of logic, or at least the philosophy of logic, is to say something about what logical consequence is. As noted above, a number of different accounts have been proposed over the years. So presumably, a logician is to come up with a theory, or theories, of what logical consequence is. Is this enterprise within the purview of anti-exceptionalism (or naturalism)? Is the goal of providing an account of what logical consequence or validity is itself a quasi-scientific matter? If so, what is the abductive methodology for this enterprise? What is it that is being explained by such an account? What is the "data" of this enterprise? Unless and until we get satisfactory answers to these questions, the thesis of anti-exceptionalism is not sufficiently articulated to take a position on it, one way or the other.

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In this section we examine the views of some of the historical figures in logic, to see the extent to which they can be classified as exceptionalists, or anti-exceptionalists. Recall, once more, the passage from Hjortland that defines the enterprise:

Logic isn't special. Its theories are continuous with science; its method continuous with scientific method. Logic isn't a priori, nor are its truths analytic truths. Logical theories are revisable, and if they are revised, they are revised on the same grounds as scientific theories. To get a feel for whether a given logician—either contemporary or historical—is an exceptionalist or anti-exceptionalist, we need some indication of the methodology they use on settling on a given logic (and, of course, a feel for the methodology of science). If our logician does not justify the particular logic chosen, or show how it compares favorably to other, rival logics, then it is probably not possible to definitively classify him or her on this front.

The founders

Let us take a brief look at the original Founder, Aristotle. As noted above, he begins with a modal characterization (*Prior Analytics*, Book 1, Chapter 2) of logical consequence:

A deduction is a discourse in which, certain things having been supposed, something different from the things supposed results of necessity because these things are so. By "because these things are so", I mean "resulting through them" and by "resulting through them", I mean "needing no further term from outside in order for the necessity to come about.

The founders

The *Prior Analytics* goes on to give a detailed account of the *forms* of various syllogisms can take, and classifies them as valid or invalid (i.e., as "deductions" or otherwise) on the basis of these forms. He shows that each of the invalid forms is invalid by giving an example, in the given form, with true premises and false conclusion. And he shows how the valid ones can be obtained from a core set of "deductions" using some rules of inference.

John Corcoran [1974] argues that Aristotle can be understood as presenting a natural deduction system, in or or less modern terms (see also Smiley [1973]).

The founders Aristotle

As far as we know, Aristotle did not contrast his account of "deduction" with a rival one—there may not have been any, or any that he knew of—and he did not mention criteria of simplicity, fruitfulness, and the like. However, with hindsight, Aristotle can be seen as following at least some of the model for logic suggested by Priest. Recall Priest [2016, §2.5]:

What plays [the] role [of data] in logic? The answer, I take it, is our intuitions about the validity or otherwise of vernacular inferences.

The founders Aristotle

Aristotle does indeed give us a method of classifying arguments in the "vernacular", and he shows that his account gets the ones that are formulated in the indicated language right. So, tentatively, he is our first anti-exceptionalist. Of course, there is a good dose of anachronism here, since nothing like scientific method was articulated then.